# Restrictions on Randomization
## Example

| | Factor A | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | | 2 | | |
| | Factor B | | | Factor B | | |
| Factor C | 1 | 2 | 3 | 1 | 2 | 3 |
| 1 | 7 | 1 | 5 | 1 | 3 | 9 |
| 2 | 6 | 3 | 8 | 4 | 2 | 6 |
| 3 | 2 | 9 | 4 | 7 | 5 | 8 |

# Restrictions on Randomization
## The Restriction Error

Linear Additive Model:

$$Y_{ijk} = \mu + A_i + \delta_{(i)} + B_j + AB_{ij} + C_k + AC_{ik} + BC_{jk} + ABC_{ijk}$$

Restriction Error:

- Follows term where restriction occurs
- Nested within restricted factor
- Considered Random with 0 df
- Affects EMS – prevents direct test on restricted factor
- A teaching device – purpose is to help you learn to recognize restrictions on randomization and their consequences

# Restrictions on Randomization

## Expected Mean Squares

| Source | a F i | b R j | c F k | EMS |
|---|---|---|---|---|
| $A_i$ | 0 | b | c | |
| $\delta_{(i)}$ | 1 | b | c | |
| $B_j$ | a | 1 | c | |
| $AB_{ij}$ | 0 | 1 | c | |
| $C_k$ | a | b | 0 | |
| $AC_{ik}$ | 0 | b | 0 | |
| $BC_{jk}$ | a | 1 | 0 | |
| $ABC_{ijk}$ | 0 | 1 | 0 | |

# Restrictions on Randomization

## Expected Mean Squares

| Source | a F i | b R j | c F k | EMS |
|---|---|---|---|---|
| $A_i$ | 0 | b | c | $\sigma^2 + c\sigma^2_{AB} + bc\sigma^2_{\delta} + bc\Phi(A)$ |
| $\delta_{(i)}$ | 1 | b | c | $\sigma^2 + c\sigma^2_{AB} + bc\sigma^2_{\delta}$ |
| $B_j$ | a | 1 | c | $\sigma^2 + ac\sigma^2_{B}$ |
| $AB_{ij}$ | 0 | 1 | c | $\sigma^2 + c\sigma^2_{AB}$ |
| $C_k$ | a | b | 0 | $\sigma^2 + a\sigma^2_{BC} + ab\Phi C$ |
| $AC_{ik}$ | 0 | b | 0 | $\sigma^2 + \sigma^2_{ABC} + b\Phi AC$ |
| $BC_{jk}$ | a | 1 | 0 | $\sigma^2 + a\sigma^2_{BC}$ |
| $ABC_{ijk}$ | 0 | 1 | 0 | $\sigma^2 + \sigma^2_{ABC}$ |

# Restrictions on Randomization

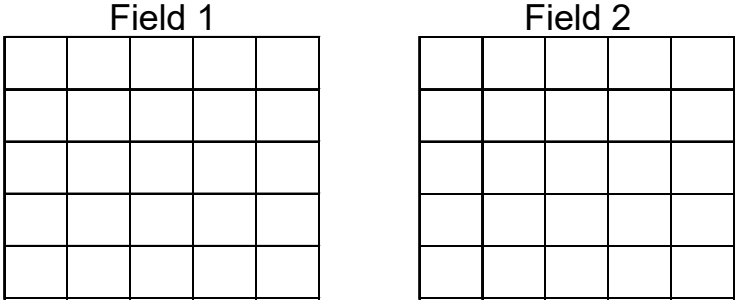
## Replicated Field Trial Example

Factors:

| | |
|---|---|
| Field (F) | 2 |
| Nitrogen (N) | 5 |
| Plot | 5 |

# Restrictions on Randomization

## Replicated Field Trial Example



# Restrictions on Randomization

## Replicated Field Trial Example

Layout:

| | Field | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | | | 2 | | | | |
| Fertilizer | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| Plot | 1 | 6 | . | . | . | 26 | 31 | . | . | . |
| | 2 | 7 | . | . | . | 27 | . | . | . | . |
| | 3 | 8 | . | . | . | . | . | . | . | . |
| | 4 | 9 | . | . | 24 | . | . | . | . | 49 |
| | 5 | 10 | . | 20 | 25 | . | . | . | 45 | 50 |

# Restrictions on Randomization

## Replicated Field Trial Example

$$Y_{ijk} = \mu + F_i + \delta_{(i)} + N_j + FN_{ij} + \varepsilon_{(ij)k}$$

| Source | 2 R i | 5 F j | 5 R k | EMS |
|---|---|---|---|---|
| $F_i$ | 1 | 5 | 5 | $\sigma^2 + 25\sigma^2_{\delta} + 25\sigma^2_{F}$ |
| $\delta_{(i)}$ | 1 | 5 | 5 | $\sigma^2 + 25\sigma^2_{\delta}$ |
| $N_j$ | 2 | 0 | 5 | $\sigma^2 + 5\sigma^2_{FN} + 10\Phi(F)$ |
| $FN_{ij}$ | 1 | 0 | 5 | $\sigma^2 + 5\sigma^2_{FN}$ |
| $\varepsilon_{(ij)k}$ | 1 | 1 | 1 | $\sigma^2$ |

# Restrictions on Randomization

## Replicated Field Trial Example

```
proc anova;
  class field nitrogen;
  model yield = field nitrogen
                field*nitrogen;
  test h=nitrogen e=field*nitrogen;
  means nitrogen / lsd e=field*nitrogen;
run;
```

```
proc mixed;
  class field nitrogen;
  model yield = nitrogen;
  random field field*nitrogen;
  lsmeans nitrogen / pdiff;
run;
```

# Restrictions on Randomization

## Greenhouse Example

Treatments:

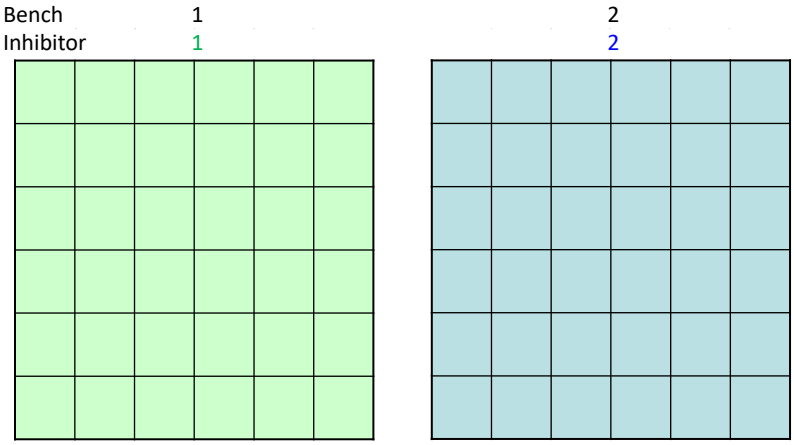
| | |
|---|---|
| Enzyme Inhibitor | 2 |
| Genotype | 6 |
| Pots | 6 |

Enzyme inhibitor treatments were placed on separate benches in the greenhouse. There were 6 replicates (pots) of each genotype randomly arranged on each bench.

Where is the restriction on randomization?

# Restrictions on Randomization

## Greenhouse Example




6 pots of each genotype were randomly placed on each bench. The inhibitor treatment is confounded with bench – it is not possible to say for certain that any effect is caused by the treatment or the bench where it was placed.

# Restrictions on Randomization

## Greenhouse Example

$$Y_{ijk} = \mu + I_i + \delta_{(i)} + G_j + IG_{ij} + \varepsilon_{(ij)k}$$

| Source | 2 F i | 6 F j | 6 R k | EMS |
|---|---|---|---|---|
| $I_i$ | 0 | 6 | 6 | $\sigma^2 + 36\sigma^2_{\delta} + 36\Phi(I)$ |
| $\delta_{(i)}$ | 1 | 6 | 6 | $\sigma^2 + 36\sigma^2_{\delta}$ |
| $G_j$ | 2 | 0 | 6 | $\sigma^2 + 12\Phi(G)$ |
| $IG_{ij}$ | 0 | 0 | 6 | $\sigma^2 + 6\sigma^2_{IG}$ |
| $\varepsilon_{(ij)k}$ | 1 | 1 | 1 | $\sigma^2$ |

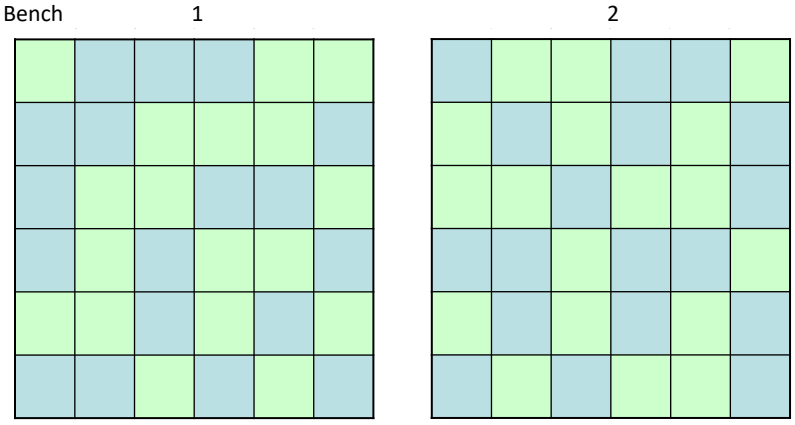
∴ Because of the restriction there is no test for the inhibitor treatment.

# Restrictions on Randomization
## Greenhouse Example

Using the same resources, how could the design (allocation of treatments to eus) be changed to provide a test for inhibitor (I)?

?

# Restrictions on Randomization
## Greenhouse Example




Place 3 pots of each inhibitor x genotype treatment combination randomly on each bench. The inhibitor treatment is no longer confounded with bench. There is still a restriction on randomization associated with bench. However, this is not a problem since you are not interested in the effect of bench per se.

# Restrictions on Randomization

## Greenhouse Example

$$Y_{ijkl} = \mu + B_i + \delta_{(i)} + I_j + BI_{ij} + G_k + BG_{ik} + IG_{jk} + BIG_{ijk} + \varepsilon_{(ijk)l}$$

| Source | 2 R i | 2 F j | 6 F k | 3 R l | EMS |
|---|---|---|---|---|---|
| $B_i$ | 1 | 2 | 6 | 3 | $\sigma^2 + 36\sigma^2_{\delta} + 36\sigma^2_{BG}$ |
| $\delta_{(i)}$ | 1 | 2 | 6 | 3 | $\sigma^2 + 36\sigma^2_{\delta}$ |
| $I_j$ | 1 | 0 | 6 | 3 | $\sigma^2 + 18\sigma^2_{BI} + 18\Phi(I)$ |
| $BI_{ij}$ | 1 | 0 | 6 | 3 | $\sigma^2 + 18\sigma^2_{BI}$ |
| $G_k$ | 2 | 2 | 0 | 3 | $\sigma^2 + 6\sigma^2_{BG} + 12\Phi(G)$ |
| $BG_{ik}$ | 1 | 2 | 0 | 3 | $\sigma^2 + 6\sigma^2_{BG}$ |
| $IG_{jk}$ | 2 | 0 | 0 | 3 | $\sigma^2 + 3\sigma^2_{BIG} + 6\,\Phi(IG)$ |
| $BIG_{ijk}$ | 1 | 0 | 0 | 3 | $\sigma^2 + 3\sigma^2_{BIG}$ |
| $\varepsilon_{(ijk)l}$ | 1 | 1 | 1 | 1 | $\sigma^2$ |

# Restrictions on Randomization

## Growth Chamber Example

Layout:

| | Temperature | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | | | | | | 2 | | | | | | | |
| **Chamber** | 1 | | | | 2 | | | | 3 | | | | 4 | | | |
| **Soil Treatment** | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| **Pot** | 1 | 6 | 11 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 56 | 61 | 66 | 71 | 76 |
| | 2 | 7 | 12 | 17 | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 | 67 | 72 | 77 |
| | 3 | 8 | 13 | 18 | 23 | 28 | 33 | 38 | 43 | 48 | 53 | 58 | 63 | 68 | 73 | 78 |
| | 4 | 9 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 69 | 74 | 79 |
| | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |

# Restrictions on Randomization
## Growth Chamber Example

Linear additive model:

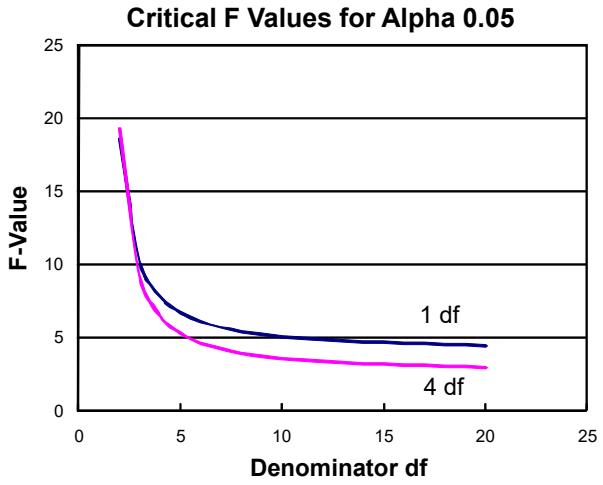
$$Y_{ijk} = \mu + T_i + C_{(i)j} + \delta_{(ij)} + S_k + TS_{ik} + CS_{(i)jk} + P_{(ijk)l}$$

# Restrictions on Randomization
## Growth Chamber Example

| Source | df | 2 F i | 2 R j | 4 F k | 5 R l | Expected Mean Square |
|---|---|---|---|---|---|---|
| $T_i$ | 1 | 0 | 2 | 4 | 5 | $\sigma^2_P + 20\sigma^2_\delta + 20\sigma^2_C + 40\Phi[T]$ |
| $C_{(i)j}$ | 2 | 1 | 1 | 4 | 5 | $\sigma^2_P + 20\sigma^2_\delta + 20\sigma^2_C$ |
| $\delta_{(ij)}$ | 0 | 1 | 1 | 4 | 5 | $\sigma^2_P + 20\sigma^2_\delta$ |
| $S_k$ | 3 | 2 | 2 | 0 | 5 | $\sigma^2_P + 5\sigma^2_{CS} + 20\Phi[S]$ |
| $TS_{ik}$ | 3 | 0 | 2 | 0 | 5 | $\sigma^2_P + 5\sigma^2_{CS} + 10\Phi[TS]$ |
| $CS_{(i)jk}$ | 6 | 1 | 1 | 0 | 5 | $\sigma^2_P + 5\sigma^2_{CS}$ |
| $P_{(ijk)l}$ | 64 | 1 | 1 | 1 | 1 | $\sigma^2_P$ |

# Restrictions on Randomization

## Growth Chamber Example




# Restrictions on Randomization

## Growth Chamber Example

SAS code:

```
proc glm;
  class temp cham soil;
  model yield = temp cham(temp) soil
                temp*soil soil*cham(temp);
  test h=temp e=cham(temp);
  test h=soil temp*soil e=soil*cham(temp);
run;
```